

# SIGNALS AND SYSTEMS IN BIOMEDICAL ENGINEERING

Signal Processing and  
Physiological Systems Modeling

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Signals and Systems in Biomedical Engineering:  
Signal Processing and Physiological Systems Modeling  
Suresh R. Devasahayam

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# SIGNALS AND SYSTEMS IN BIOMEDICAL ENGINEERING

## Signal Processing and Physiological Systems Modeling

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## Series Preface

In the past few years Biomedical Engineering has received a great deal of attention as one of the emerging technologies in the last decade and for years to come, as witnessed by the many books, conferences, and their proceedings. Media attention, due to the applications-oriented advances in Biomedical Engineering, has also increased. Much of the excitement comes from the fact that technology is rapidly changing and new technological adventures become available and feasible every day. For many years the physical sciences contributed to medicine in the form of expertise in radiology and slow but steady contributions to other more diverse fields, such as computers in surgery and diagnosis, neurology, cardiology, vision and visual prosthesis, audition and hearing aids, artificial limbs, biomechanics, and biomaterials. The list goes on.

It is therefore hard for a person unfamiliar with a subject to separate the substance from the hype. Many of the applications of Biomedical Engineering are rather complex and difficult to understand even by the not so novice in the field. Much of the hardware and software tools available are either too simplistic to be useful or too complicated to be understood and applied. In addition, the lack of a common language between engineers and computer scientists and their counterparts in the medical profession, sometimes becomes a barrier to progress.

This series of books is initiated with the above in mind: it addresses the biomedical engineer, the students of biomedical engineering, the computer scientist, and any other technically oriented person who wants to learn about Biomedical Engineering, as well as anyone who wants to solve problems dealing with health, medicine, and engineering. It addresses the physician and the health professional as well, introducing them to engineering jargon. Medical practitioners face problems that need solutions, yet technological

advances and their complexity leave most of them in awe. The engineer, physicist, or computer scientist on the other hand does not have the medical knowledge needed to solve the problems at hand all of the time. It is through books, like the ones in this series, that the gap can be bridged, a common understanding of the problems can be achieved, and solutions come to light.

This series aims to attract books on topics that have not been addressed; by experts in the field that never had an incentive to write, books that will extend and complement the ones in existence.

We hope to bring this synergy to fruition through the books in the series.

*Evangelia Micheli-Tzanakou*  
*Series Editor*

## Preface

Biomedical Signal Processing involves the use of signal processing techniques for the interpretation of physiological measurements and the understanding of physiological systems. Although the analytical techniques of signal processing are obtained largely from developments in telecommunications and applied mathematics, the nature of physiological data requires substantial biological understanding for its interpretation. It is a well recognized idea that every instance of the use of signal processing techniques is predicated on explicit or implicit models. In the case of physiological data the interpretation of the data contains certain assumptions about the underlying physiological processes which we may call the model of the system. Whether one uses a model that corresponds to physical and chemical entities (a biophysical model) or simply a model defining an input-output relationship (a black-box model) the assumed model determines the nature of noise reduction or feature extraction that is performed.

The lecture notes that have formed this book were written for courses that I taught at IIT-Bombay on Signal Processing and Physiological Systems Modeling to graduate students in Biomedical Engineering. These courses have evolved over the years and at present they are taught over 1½ semesters in two courses called Signals and Systems, and Physiological Systems Modeling which may be regarded a single 1½ semester course. The class comprises students with engineering degrees as well as students with medical degrees. Therefore, it was something of a challenge to structure the course so that all the students would find it sufficiently engaging. The aim of the course is to introduce the students to physiological signal analysis with explicit understanding of the underlying conceptual models. The measurable goal of the course is to see that students can read a typical paper published in

the standard journals of biomedical engineering. Therefore, the last couple of weeks of this course consists of discussing two or three recent papers on physiological modeling and signal analysis. Although a number of books are available on signal processing, including several on Biomedical Signal Processing, I found that no single book or even a small set of books could satisfactorily serve as a text for this course. My solution was to use several books as reference texts supplemented with lecture notes and journal papers. I gave the students detailed programming exercises so that their understanding of the material would be firmly established. In the mid 1990's I found that my dilemma of a suitable text for Biomedical Signal Analysis was shared by many others, evidenced by the publication of several books on "Biomedical Signal Processing". However, these books treated the subject as a specialization of Signal Processing and Electronics Communications. In my opinion this subtracted from the principal biomedical engineering enterprise of being an interdisciplinary program which recognized the importance of model-based data interpretation. Therefore, my lecture notes grew with advances in the subject.

Beginning with a broad introduction to signals and systems the book proceeds to contemporary techniques in digital signal processing. While maintaining continuity of mathematical concepts, the emphasis is on practical implementation and applications. The book only presumes knowledge of college mathematics and is suitable for a beginner in the subject; however, a student with a previous course in analog and digital signal processing will find that less than a third of the book contains a bare treatment of classical signal processing.

Not surprisingly a lot of the examples and models that I use in teaching are informed by my own interests in skeletal muscle physiology and electrophysiology. Therefore, some of the modeling of muscles and myoelectric activity arose from data collected in my experimental work. In this book most of the diagrams of myoelectric activity are real data recorded in my lab; on the other hand, although I have recorded muscle force from isolated muscles, single motor units and also on human volunteers, I chose to use only simulated force data in this book. I have also expanded on models of other physiological systems mentioned in the literature to introduce the student to the rich variety of experimental and analytical techniques used in the study of physiological systems.

*Suresh Devasahayam*

## Acknowledgments

I would first like to thank my proximate critics, the students who took my courses in physiological systems modeling and signal analysis; their questions and comments were invaluable in the enrichment of my own understanding and in the development of this book. I am indebted to my colleagues at IIT-Bombay for lively exchange of ideas in teaching and research. In particular, I would like to thank Rohit Manchanda with whom I have taught several courses over the last decade; his critical suggestions have been invaluable in the enhancement of my teaching methods including a lot of the material in this book. I would also like to thank Vikram Gadre for many discussions on signal processing. The evolution of my course material owes no small debt to the flexibility that I was allowed in designing courses and teaching them, for which I must thank Dr. Subir Kar and Dr. Rakesh Lal. I also wish to acknowledge the support received from the Ministry of Human Resources Development and the All India Council for Technical Education for projects that contributed to the development of teaching material. My specific interest in experimental physiology and signal analysis was molded by my graduate advisor Dr. Sandercock at the University of Illinois at Chicago who introduced me to physiological research and encouraged my ever widening interest in the subject.

Moving to the more personal, I wish to mention Andrew Krylow not only because he shared with me his youthful and exuberant love for scientific inquiry during my graduate student days, but also because his untimely death in 1998 has indeed bereaved me.

My interest in engineering and science I owe to both my parents who taught me by example the pleasure of working with my hands and using tools; my father also imparted to me his fascination with all gadgets and the

idea that they can be dismantled, understood and often fixed. I remember with fondness my grandfather whose desire to inculcate a love of reading rivaled my parents' and who knew most of my mathematics books more intimately than I did.

It is a pleasure to acknowledge the contribution of all these people to this work. I must, of course, add that the responsibility for any errors in this book is entirely mine.

There are also many others who have contributed not only directly to the contents of this book but also to the broader circumstances of writing it. In this respect my acknowledgment here is quite incomplete.

*SRD*

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## Chapter 1

# Introduction to Systems Analysis and Numerical Methods

### 1.1 The Systems Approach to Physiological Analysis

The recording and analysis of physiological signals has permeated all aspects of the study of biological organisms, from basic science to clinical diagnosis. For example, the clinical recording of various biopotential signals has become an essential component in the diagnosis of all organs involving excitable tissue; information from pressure and flow signals is an important part of cardiovascular care; even in the diagnosis of digestive and excretory disorders signal analysis provides valuable assistance. Implicit in the analysis of these signals is an understanding of the mechanisms involved in their physiological generation. It is important to note that even if an explicit model is not postulated the very act of using an analytic procedure implies a certain model for the physiological process. This fact is often ignored to the detriment of the analysis. When explicit models are postulated every measurement acts to either support it or weaken it. In fact, analyses predicated on explicitly defined models make for good science in that clear testable hypotheses are available.

Physiological modeling involves the development of mathematical, electrical, chemical or other analogues whose behavior closely approximates the behavior of a particular physiological system. It is, of course, desirable that every aspect of the model corresponds to features of the physiological system under consideration. Such models may be called biophysical models. However, in most physiological systems only a few of the features are observable. Therefore, a model based on empirical relations between these observable features often has as much utility as a detailed physical one. Such models may be termed black-box models since they make no attempt to

describe the internal mechanisms of these systems. To clinical practitioners whose main interest is in the classification of the status of any physiological system as normal or pathological, the black box model is not only sufficient but may even be preferable on account of its relative simplicity. However, for physiologists and other basic scientists, biophysical models are more interesting. It is, of course, very satisfying to the scientist when biophysical models can be shown to be reducible to utilitarian black-box models. We shall concentrate on black-box modeling using techniques of linear-systems analysis. However, the links between physiological components and model aspects are also emphasized and some physiological models are discussed in reasonable detail. In some cases the reduction of biophysical models to linear systems models is also shown.

### **1.1.1 Physiological Signals and Systems**

Any physical quantity that varies as a function of an independent variable can be considered a signal. The independent variable is usually time, although not always so; space or distance is also frequently used as the independent variable. The variation of potentials, mechanical force or position, pressure, volume, etc., as a function of time are all commonly used physiological signals. These signals are generated by physiological processes following well defined physical laws. All physiological systems accept various inputs from other organs, the external environment, etc., and produce outputs in response to these stimuli. This concept of a physiological system producing one or more outputs in response to one or more inputs is the basic idea of systems modeling. The input-output relation is characteristic of the system. There are mathematical procedures for characterizing signals using analytical functions. We can also mathematically describe a system that acts on a set of input functions to produce a set of output functions. If the input and output are measured, then, the system characteristics can be estimated. Such estimation of the system characteristics when performed under diseased states as well as normal conditions help to characterize the disease quantitatively in comparison to the normal state. Alternatively, given the system description it should be possible to predict the system output for any arbitrary set of inputs. Thus the system description allows estimation of the output given the input or conversely estimation of the input if the output is known. The primary task in signal analysis is to characterize signals and the systems that produce or process them.

### 1.1.2 Linear Systems Modeling in Physiology

In general most real systems have complex properties and no simple characterization is possible. However, it is usually convenient to restrict the study of systems to some limited conditions where the system may be said to be *linear*. In this book we shall see some of the conditions under which physiological systems do and do not submit to linear systems analysis. Although we shall concentrate on analytical tools for linear systems, it is extremely important to understand when a linear model may be inadequate and even misleading. The advantage of linear systems analysis is that a very large set of analytical tools are available. Nonlinear techniques not only have to be tailored specifically to each situation, but also tend to be more complex and computationally tedious than linear techniques.

## 1.2 Numerical Methods for Data Analysis and Simulation

The availability of cheap computing power makes a lot of models available easily to physiologists and medical practitioners. Modern computers have not only good computational capabilities but also very good graphical displays, thereby making the output of models convenient for non-mathematical users. Of course, graphical presentation in itself uses visual analogy for physical behavior. Since modern computers are all discrete numerical machines while physiological systems are fundamentally continuous, some approximations are required in order to use discrete modeling for continuous-time systems. The issue of discretization is dealt with in some detail in the early part of the book, and subsequently a number of digital techniques for the analysis of signals and systems are discussed.

The earliest models of physiological system were physical analogies. Even now many students in high school are introduced to the ideas of respiration and blood flow using physical models involving air flow and water in tubes respectively. Physical models are useful in extending intuitive knowledge in one area to another. However, physical models are limited by constraints of implementation to rather simple systems. Mathematical models are also similarly limited by our ability to solve the necessary sets of equations, but this limitation is much less confining than the physical construction of analogous models.

Most mathematical descriptions of physiological systems use differential equations. Therefore, the analysis of these systems requires solving differential equations. Such solutions of differential equations can in principle be done analytically (i.e., on paper), physically (i.e., by building a physical analogue), or numerically (i.e., on a digital computer). In the early

days of computational models differential equations were solved using analogue computers. The analogue computers were electronic circuits whose behavior mimicked that of the system being modeled. Modern computer models use digital computers to solve the system equations. Contemporary digital computers are Turing machines that do not physically imitate the system behavior but instead “run programs” that make the general purpose computer imitate the system being modeled. A very important aspect of digital computers is that they inherently require *discretized* representations of the modeled system. This places some important constraints on the modeling process. The constraints are exemplified in the numerical implementation of integration and differentiation which are basic components of systems models.

### 1.2.1 Numerical Integration and Differentiation

A continuous function of time can be discretized by using a sequence of numbers corresponding to the value of the function at discrete points in time. Since most systems can be described using differential equations, numerical differentiation and integration are fundamental to computer implementation of mathematical models of systems.

The derivative of the function at a point in time  $t$  can be calculated from the discretized function in more than one way. By definition:

$$\frac{dx(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} \quad (1.1)$$

where  $\Delta x$  is the change in  $x(t)$  over the time interval  $\Delta t$ . However, when differentiation is implemented on a digital computer  $\Delta t$  is necessarily finite. In the limit  $\Delta t \rightarrow 0$  we can write

$$\begin{aligned} \Delta x &= [x(t + \Delta t) - x(t)] \\ &= [x(t + \Delta t / 2) - x(t - \Delta t / 2)] \\ &= [x(t) - x(t - \Delta t)] \end{aligned} \quad (1.2)$$

If  $\Delta t$  is finite these are only approximations. Therefore, using a digital computer only an approximation to  $dx/dt$  is possible. In practice, with a finite discretization interval  $T = \Delta t$  the following approximation may be used:

$$\frac{dx(t)}{dt} \approx \frac{x(t) - x(t - T)}{T} \quad (1.3)$$