

# Portfolio Theory & Financial Analyses

Robert Alan Hill



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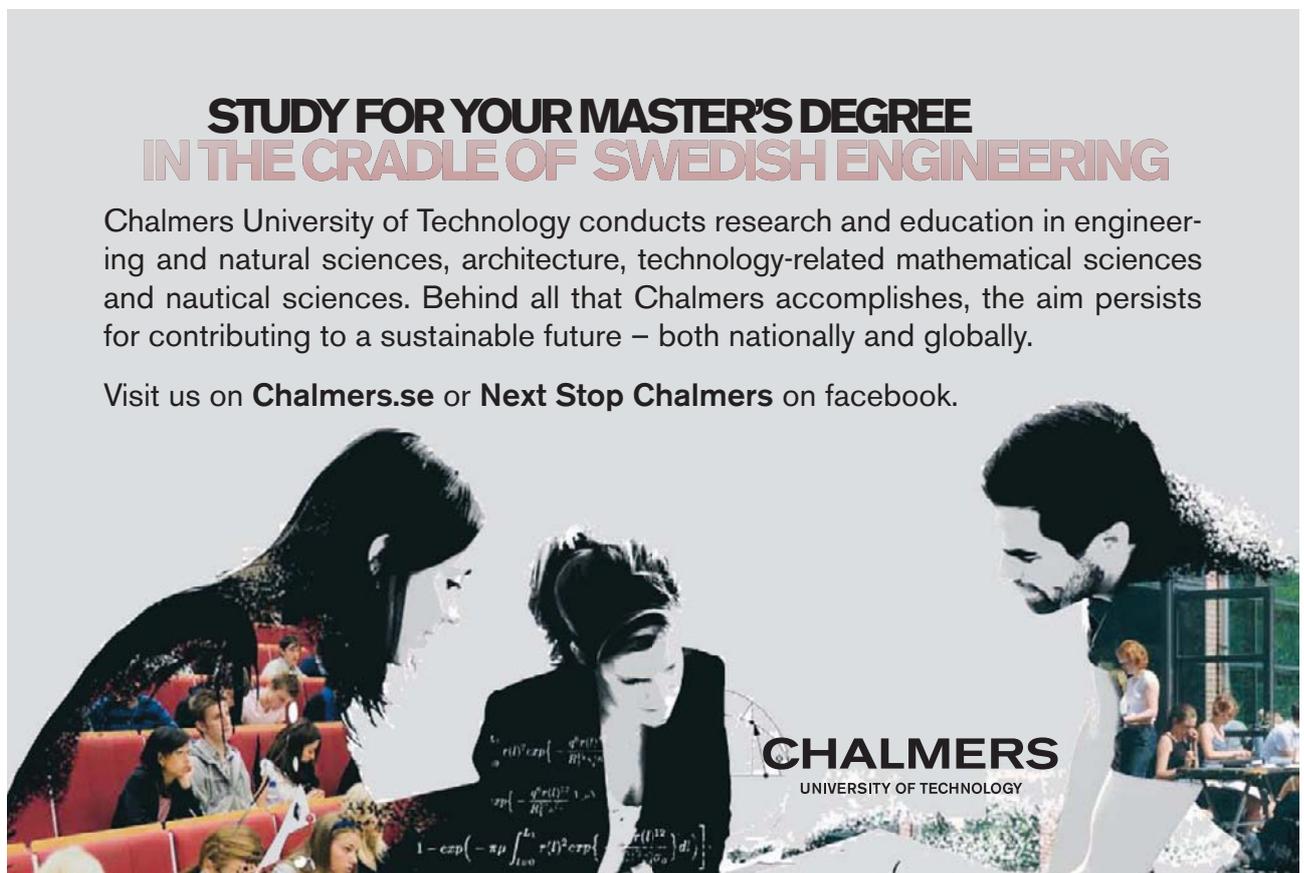


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# About the Author

With an eclectic record of University teaching, research, publication, consultancy and curricula development, underpinned by running a successful business, Alan has been a member of national academic validation bodies and held senior external examinerships and lectureships at both undergraduate and postgraduate level in the UK and abroad.

With increasing demand for global e-learning, his attention is now focussed on the free provision of a financial textbook series, underpinned by a critique of contemporary capital market theory in volatile markets, published by bookboon.com.

To contact Alan, please visit Robert Alan Hill at [www.linkedin.com](http://www.linkedin.com).



# **Part I:** An Introduction

# 1 An Overview

## Introduction

Once a company issues shares (common stock) and receives the proceeds, it has no *direct* involvement with their subsequent transactions on the capital market, or the price at which they are traded. These are matters for negotiation between existing shareholders and prospective investors, based on their own financial agenda.

As a basis for negotiation, however, the company plays a pivotal *agency* role through its implementation of investment-financing strategies designed to maximise profits and shareholder wealth. What management do to satisfy these objectives and how the market reacts are ultimately determined by the law of supply and demand. If corporate returns exceed market expectations, share price should rise (and vice versa).

But in a world where ownership is divorced from control, characterised by economic and geo-political events that are also beyond management's control, this invites a question.

How do companies determine an optimum portfolio of investment strategies that satisfy a multiplicity of shareholders with different wealth aspirations, who may also hold their own diverse portfolio of investments?

## 1.1 The Development of Finance

As long ago as 1930, Irving Fisher's *Separation Theorem* provided corporate management with a lifeline based on what is now termed Agency Theory.

He acknowledged implicitly that whenever ownership is divorced from control, direct communication between management (*agents*) and shareholders (*principals*) let alone other stakeholders, concerning the likely profitability and risk of every corporate investment and financing decision is obviously impractical. If management were to implement optimum strategies that satisfy each shareholder, the company would also require prior knowledge of every investor's stock of wealth, dividend preferences and risk-return responses to their strategies.

According to Fisher, what management therefore, require is a model of *aggregate* shareholder behaviour. A theoretical abstraction of the real world based on simplifying assumptions, which provides them with a methodology to communicate a diversity of corporate wealth maximising decisions.

To set the scene, he therefore assumed (not unreasonably) that all investor behaviour (including that of management) is *rational* and *risk averse*. They prefer high returns to low returns but less risk to more risk. However, risk aversion does not imply that rational investors will not take a chance, or prevent companies from retaining earnings to gamble on their behalf. To accept a higher risk they simply require a commensurately higher return, which Fisher then benchmarked.

Management's minimum rate of return on incremental projects financed by retained earnings should equal the return that existing shareholders, or prospective investors, can earn on investments of equivalent risk elsewhere.

He also acknowledged that a company's acceptance of projects internally financed by retentions, rather than the capital market, also denies shareholders the opportunity to benefit from current dividend payments. Without these, individuals may be forced to sell part (or all) of their shareholding, or alternatively borrow at the market rate of interest to finance their own preferences for consumption (income) or investment elsewhere.

To circumvent these problems Fisher assumed that if capital markets are *perfect* with no barriers to trade and a free flow of information (more of which later) a firm's *investment* decisions can not only be *independent* of its shareholders' *financial* decisions but can also satisfy their wealth maximisation criteria.

In Fisher's perfect world:

- Wealth maximising firms should determine optimum *investment* decisions by *financing* projects based on their *opportunity* cost of capital.
- The *opportunity cost* equals the *return* that existing shareholders, or prospective investors, can earn on investments of equivalent risk elsewhere.
- Corporate projects that earn rates of return less than the opportunity cost of capital should be rejected by management. Those that yield equal or superior returns should be accepted.
- Corporate earnings should therefore be distributed to shareholders as dividends, or retained to fund new capital investment, depending on the relationship between project profitability and capital cost.
- In response to rational managerial dividend-retention policies, the final consumption-investment decisions of rational shareholders are then determined independently according to their personal preferences.
- In perfect markets, individual shareholders can always borrow (lend) money at the market rate of interest, or buy (sell) their holdings in order to transfer cash from one period to another, or one firm to another, to satisfy their income needs or to optimise their stock of wealth.

**Activity 1**

Based on Fisher's Separation Theorem, share price should rise, fall, or remain stable depending on the inter-relationship between a company's project returns and the shareholders desired rate of return. Why is this?

For detailed background to this question and the characteristics of perfect markets you might care to download "Strategic Financial Management" (both the text and exercises) from [bookboon.com](http://bookboon.com) and look through their first chapters.

## 1.2 Efficient Capital Markets

According to Fisher, in perfect capital markets where ownership is divorced from control, the separation of corporate dividend-retention decisions and shareholder consumption-investment decisions is not problematical. If management select projects using the shareholders' desired rate of return as a cut-off rate for investment, then at worst corporate wealth should stay the same. And once this information is communicated to the outside world, share price should not fall.

Of course, the Separation Theorem is an abstraction of the real world; a model with questionable assumptions. Investors do not always behave rationally (some speculate) and capital markets are not perfect. Barriers to trade do exist, information is not always freely available and not everybody can borrow or lend at the same rate. But instead of asking whether these assumptions are divorced from reality, the relevant question is whether the model provides a sturdy framework upon which to build.

Certainly, theorists and analysts believed that it did, if Fisher's impact on the subsequent development of finance theory and its applications are considered. So much so, that despite the recent global financial meltdown (or more importantly, because the events which caused it became public knowledge) it is still a basic tenet of finance taught by Business Schools and promoted by other vested interests world-wide (including governments, financial institutions, corporate spin doctors, the press, media and financial web-sites) that:

Capital markets may not be *perfect* but are still reasonably *efficient* with regard to how analysts *process information* concerning corporate activity and how this changes market values once it is conveyed to investors.

An efficient market is one where:

- Information is universally available to all investors at a low cost.
- Current security prices (debt as well as equity) reflect all relevant information.
- Security prices only change when new information becomes available.

Based on the pioneering research of Eugene Fama (1965) which he formalised as the “efficient market hypothesis” (EMH) it is also widely agreed that *information processing efficiency* can take *three forms* based on *two types* of analyses.

*The weak form* states that *current* prices are determined solely by a *technical* analysis of *past* prices. Technical analysts (or *chartists*) study historical price movements looking for cyclical patterns or trends likely to repeat themselves. Their research ensures that significant movements in current prices relative to their history become widely and quickly known to investors as a basis for immediate trading decisions. Current prices will then move accordingly.

*The semi-strong form* postulates that current prices not only reflect price history, but all *public* information. And this is where *fundamental analysis* comes into play. Unlike chartists, *fundamentalists* study a company and its business based on historical records, plus its current and future performance (profitability, dividends, investment potential, managerial expertise and so on) relative to its competitive position, the state of the economy and global factors.

In weak-form markets, fundamentalists, who make investment decisions on the expectations of individual firms, should therefore be able to “out-guess” chartists and profit to the extent that such information is not assimilated into past prices (a phenomenon particularly applicable to companies whose financial securities are infrequently traded). However, if the semi-strong form is true, fundamentalists can no longer gain from their research.

*The strong form* declares that current prices fully reflect *all information*, which not only includes all publically available information but also *insider* knowledge. As a consequence, unless they are lucky, even the most privileged investors cannot profit in the long term from trading financial securities before their price changes. In the presence of strong form efficiency the market price of any financial security should represent its intrinsic (true) value based on anticipated returns and their degree of risk.

So, as the EMH strengthens, speculative profit opportunities weaken. Competition among large numbers of increasingly well-informed market participants drives security prices to a consensus value, which reflects the best possible forecast of a company's uncertain future prospects.

Which strength of the EMH best describes the capital market and whether investors can ever “beat the market” need not concern us here. The point is that whatever levels of efficiency the market exhibits (weak, semi- strong or strong):

- Current prices reflect all the relevant information used by that market (price history, public data and insider information, respectively).
- Current prices only change when new information becomes available.

It follows, therefore that prices must follow a “random walk” to the extent that new information is *independent* of the last piece of information, which they have already absorbed.

- And it this phenomenon that has the most important consequences for how management model their strategic investment-financing decisions to maximise shareholder wealth

#### Activity 2

Before we continue, you might find it useful to review the Chapter so far and briefly summarise the main points..

### 1.3 The Role of Mean-Variance Efficiency

We began the Chapter with an idealised picture of investors (including management) who are rational and risk-averse and formally analyse one course of action in relation to another. What concerns them is not only profitability but also the likelihood of it arising; a *risk-return* trade-off with which they feel comfortable and that may also be unique.

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Thus, in a sophisticated mixed market economy where ownership is divorced from control, it follows that the objective of strategic financial management should be to implement optimum investment-financing decisions using risk-adjusted wealth maximising criteria, which satisfy a multiplicity of shareholders (who may already hold a diverse portfolio of investments) by placing them all in an equal, optimum financial position.

No easy task!

But remember, we have not only assumed that investors are rational but that capital markets are also reasonably efficient at processing information. And this greatly simplifies matters for management. Because today's price is *independent* of yesterday's price, efficient markets have *no memory* and individual security price movements are *random*. Moreover, investors who comprise the market are so large in number that no one individual has a comparative advantage. In the short run, "you win some, you lose some" but long term, investment is a *fair game* for all, what is termed a "martingale". As a consequence, management can now afford to take a *linear* view of investor behaviour (as new information replaces old information) and model its own plans accordingly.

What rational market participants require from companies is a diversified investment portfolio that delivers a maximum return at minimum risk.

What management need to satisfy this objective are investment-financing strategies that maximise corporate wealth, validated by simple *linear* models that statistically quantify the market's risk-return *trade-off*.

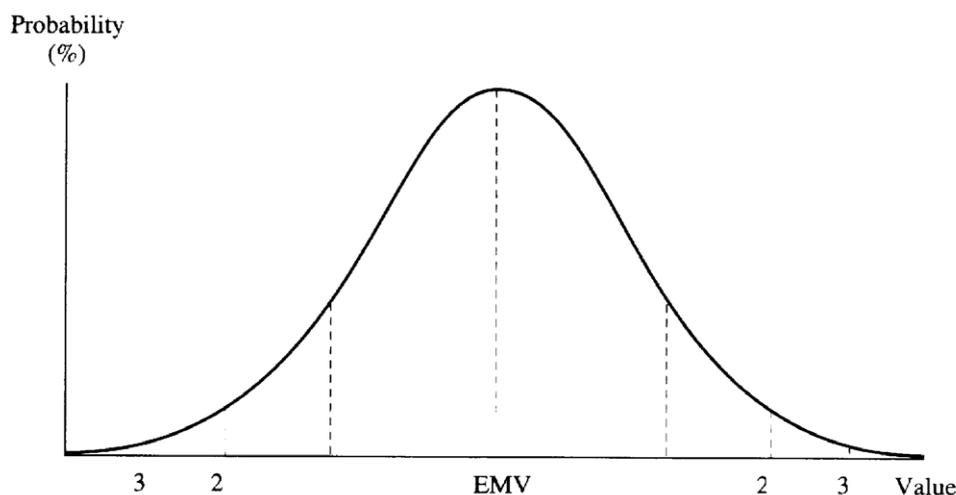
Like Fisher's Separation Theorem, the concept of linearity offers management a lifeline because in *efficient* capital markets, rational investors (including management) can now assess anticipated investment returns ( $r_i$ ) by reference to their probability of occurrence, ( $p_i$ ) using classical statistical theory.

If the returns from investments are assumed to be *random*, it follows that their *expected return* ( $R$ ) is the expected monetary value (EMV) of a symmetrical, *normal* distribution (the familiar "bell shaped curve" sketched overleaf). Risk is defined as the *variance* (or dispersion) of individual returns: the greater the variability, the greater the risk.

Unlike the mean, the statistical measure of dispersion used by the market or management to assess risk is partly a matter of convenience. The *variance* (VAR) or its square root, the *standard deviation* ( $\sigma = \sqrt{\text{VAR}}$ ) is used.

When considering the *proportion* of risk due to some factor, the variance ( $\text{VAR} = \sigma^2$ ) is sufficient. However, because the standard deviation ( $\sigma$ ) of a normal distribution is measured in the same units as (R) the expected value (whereas the variance ( $\sigma^2$ ) only summates the squared deviations around the mean) it is more convenient as an *absolute* measure of risk.

Moreover, the standard deviation ( $\sigma$ ) possesses another attractive statistical property. Using confidence limits drawn from a Table of *z* statistics, it is possible to establish the *percentage probabilities* that a random variable lies within *one, two or three standard deviations above, below or around* its expected value, also illustrated below.



**Figure 1.1:** The Symmetrical Normal Distribution, Area under the Curve and Confidence Limits

Armed with this statistical information, investors and management can then accept or reject investments according to the degree of confidence they wish to attach to the likelihood (risk) of their desired returns. Using decision rules based upon their optimum criteria for *mean-variance efficiency*, this implies management and investors should pursue:

- Maximum expected return (R) for a given level of risk, (s).
- Minimum risk (s) for a given expected return (R).

Thus, our conclusion is that if modern capital market theory is based on the following three assumptions:

- (i) Rational investors,
- (ii) Efficient markets,
- (iii) Random walks.

The normative wealth maximisation objective of strategic financial management requires the optimum selection of a portfolio of investment projects, which maximises their expected return (R) commensurate with a degree of risk (s) acceptable to existing shareholders and potential investors.

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